Simplifying Negations Math 180

Name: Solutions Section: _____

1. Negate the following statement, justifying each step

$$(P \wedge \neg Q)$$

$\neg(P \wedge \neg Q) \equiv \neg P \vee \neg(\neg Q)$ De Morgan

$\equiv \neg P \vee Q$ Double Negation

2. Negate the following statement, justifying each step

$$(P \vee Q) \wedge R$$

$\neg\big((P \vee Q) \wedge R\big) \equiv \neg(P \vee Q) \vee \neg R$ De Morgan

$\equiv (\neg P \wedge \neg Q) \vee \neg R$ De Morgan

3. Negate the following statement, justifying each step

$$(P \wedge \neg Q) \vee (\neg R \wedge S)$$

$\neg\big((P \wedge \neg Q) \vee (\neg R \wedge S)\big) \equiv \neg\big(P \wedge \neg Q\big) \wedge \neg(\neg R \wedge S)\big)$ De Morgan

$\equiv \big(\neg(P) \vee \neg\neg Q\big) \wedge \big(\neg\neg R \vee \neg S\big)$ De Morgan

$\equiv (\neg P \vee Q) \wedge (R \vee \neg S)$ Double Negation

4. Negate $\neg P \to \neg Q$

$\neg(\neg P \to \neg Q) \equiv \neg\big(\neg(\neg P) \vee \neg Q\big)$ or-form of $\to$

$\equiv \neg(P \vee \neg Q)$ Double Negation

$\equiv \neg P \wedge \neg\neg Q$ De Morgan

$\equiv \neg P \wedge Q$ Double Negation.

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Name: _____ Section: _____

4. Negate the following statement, justifying each step

$$\neg P \to (\neg Q \to P)$$

$\neg(\neg P \to (\neg Q \to P)) \equiv \neg(\neg\neg P \vee (\neg Q \to P))$ or-form of $\to$

$\equiv \neg(P \vee (\neg Q \to P))$ Double Negation

$\equiv \neg P \wedge \neg(\neg Q \to P)$ De Morgan

$\equiv \neg P \wedge \neg(\neg\neg Q \vee P)$ or-form of $\to$

$\equiv \neg P \wedge \neg(Q \vee P)$ Double Negation

$\equiv \neg P \wedge (\neg Q \wedge \neg P)$ DeMorgan

5. Negate the following statement, justifying each step

$$P \leftrightarrow (\neg Q \wedge R)$$

$\neg(P \leftrightarrow (\neg Q \wedge R)) \equiv \neg((P \to (\neg Q \wedge R)) \wedge ((\neg Q \wedge R) \to P))$ $\to$ form of $\leftrightarrow$

$\equiv \neg(P \to (\neg Q \wedge R)) \vee \neg((\neg Q \wedge R) \to P)$ De Morgan

$\equiv \neg(\neg P \vee (\neg Q \wedge R)) \vee \neg(\neg(\neg Q \wedge R) \vee P)$ $\vee$ form of $\to$

$\equiv (\neg\neg P \wedge \neg(\neg Q \wedge R)) \vee (\neg\neg(\neg Q \wedge R) \vee \quad)$ De Morgan

$\equiv (P \wedge (\neg\neg Q \wedge \neg R)) \vee ((\neg Q \wedge R) \vee \quad)$

$\equiv (P \wedge (Q \wedge \neg R)) \vee (\neg Q \wedge R \wedge \neg P)$

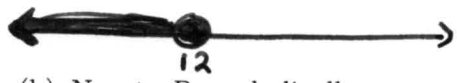
Name: _____ Section: _____

7. Consider the statement

$$R: \quad x \leq 12$$

(a) Sketch the set of x such that R is *true*. Write the set in interval notation.

R is true if x is in $(-\infty, 12]$



(b) Negate R symbolically.

$\neg(x \leq 12) \equiv \quad x > 12$

(c) Sketch the set of x so that $\neg R$ is *true*. Write the set in interval notation.

the negation is true (R is false) if x is in $(12, \infty)$

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8. Consider the statement

$$R: \quad x \leq 12 \text{ or } x > 30$$

(a) Sketch the set of x such that R is *true*. Write the set in interval notation.

R is true if x is in $(-\infty, 12] \cup (30, \infty)$

(b) Negate R symbolically.

$\neg(x \leq 12 \text{ or } x > 30) \equiv \neg(x \leq 12) \text{ and } \neg(x > 30) \equiv x > 12 \text{ and } x \leq 30$

(c) Sketch the set of x so that $\neg R$ is *true*. Write the set in interval notation.

the negation is true (R is false) if x is in $(12, 30]$

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9. Negate the statement

$$R: \quad (x < y) \rightarrow (x^2 < y^2)$$

$\neg(x < y \rightarrow x^2 < y^2) \equiv \neg(\neg(x < y) \vee x^2 < y^2)$ V-form of $\rightarrow$

$\equiv \neg\neg(x < y) \wedge \neg(x^2 < y^2)$ De Morgan

$\equiv (x < y) \wedge (x^2 \geq y^2)$

Name: _____ Section: _____

10. Consider the sentence: "If p is a polynomial and its degree is greater than 2, then p is either always increasing or always decreasing."

(a) Translate the sentence into symbols. Label all basic claims.

P: p is a poly
Q: degree is ≥ 2
R: P is always increasing
S: P is always decreasing

$(P \wedge Q) \to (R \vee S)$

(b) Negate the sentence symbolically.

$\neg((P \wedge Q) \to (R \vee S)) \equiv \neg(\neg(P \wedge Q) \vee (R \vee S))$

$\equiv \neg\neg(P \wedge Q) \wedge \neg(R \vee S) \equiv (P \wedge Q) \wedge (\neg R \wedge \neg S)$

(c) Translate the negations back into English.

P is a polynomial of degree 2
that is not always increasing
and is not always decreasing.

De Morgan's Laws

$$\neg(P \wedge Q) \equiv (\neg P) \vee (\neg Q)$$
$$\neg(P \vee Q) \equiv (\neg P) \wedge (\neg Q)$$

Or form of Implication

$$P \to Q \equiv (\neg P) \vee Q$$

Implies form of Iff

$$P \leftrightarrow Q \equiv (P \to Q) \wedge (Q \to P)$$

Double Negation

$$\neg\neg P \equiv P$$

Trichotemy: For each real number a and x,

$$(x = a) \vee (x > a) \vee (x < a)]$$
